Subjective Risk Analysis Approaches of LCP

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Abstract

This paper considers the subjective modelling of the risk events associated to Large Combustion Plants by linguistic variables. Computing with words approach is described under both fuzzy (Zadeh's fuzzy sets) and intuitionistic fuzzy (Atanassov's intuitionistic fuzzy sets) models. The described algorithms easily can be applied to subjective risk analysis.

Keywords: *LCP, risk modelling, CWW, fuzzy sets, intuitionistic fuzzy numbers*

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1. **Introduction**

During last century, important progress has been made in understanding risky events under uncertainty. A risky event is any event that is not known for sure ahead of time. To analyse risk is necessary to decompose risks into two components: probability (or likelihood) of the risk, and the impact/loss the risk can cause. For the aim of this paper the risk is measured according to: Risk = Probability x Impact. However, not all risky events are repeatable, and this is the case of risky events concerning Large Combustion Plants (LCPs) where some risky events are observed very rarely, or under changing conditions. In these cases probability is seen as "a subjective and personal evaluation of the relative likelihood of an event reflecting the individual's own information and belief", as Chavas [6] remarked.

Various models of subjective risky events have been presented in literature (Anscombe [1], Kopylov [12], to mention only a few contributions). Savage's expected utility theory is such a model of a subjective theory of probability (see Epstein & Zhang [11] for unambiguous events).

Some events can be explained in a cause-effect framework, and the usage of some adequate procedures will help to minimize the impact, but there may be cases where relative likelihood rankings by an individual are not consistent with probability rankings. As alternatives, researchers proposed the usage of fuzzy sets (Zadeh [22]), and "ambiguity theory". The most recent proposal by Smarandache [17], neutrosophic thinking, is based on the integration of models related on *likelihood* (truth), *falsehood*, and *indeterminacy*.

This paper considers the subjective risk evaluation for the management of of LCPs [9, 10]. The motivation of research is based on Directive 2001/80/EC [8] on the limitation of emissions of certain pollutants into the air from large combustion plants (the LCP Directive). According to European Commission [8], the control of emissions from large combustion plants – those whose rated thermal input is equal to or greater than 50 MW – plays an important role "to combat acidification, eutrophication and ground-level ozone as part of the overall strategy to reduce air pollution".

The next section deals with linguistic variables associated to potential hazards for LCPs, to be used in the third section for fuzzy sets associated models to LCP risk analysis. Fuzzy numbers are used both for modelling the possibility to experiment a failure and for impact description. The fourth section uses Atanassov intuitionistic fuzzy numbers [2] in order to cover the degree of non-possibility, and impact.

2. **Linguistic variables associated to potential hazards for LCPs**

In fuzzy set theory, there are used *linguistic variables* to describe the probability of risky events, which can be characterised by their fuzzy set membership functions. As Pei & Shi [16] says, "linguistic values are computational variables, which makes the results of risk analysis no loss of information and easier to communicate to decision- and policy-makers."

The following examples related to LCP risk management show the usage of linguistic variables [10]:

- The total destruction of the facilities (terrorist attack with conventional weapons or nuclear) – *extremely low probability of occurrence*, *major impact*;
- The cracking or breaking of storage tanks and leaking hydrochloric acid content (terrorist attack or mechanical hazard due to an earthquake, accidental bumping, accidentally breaking sockets, defective materials etc) – *low probability of occurrence*, *significant impact*;
- The cracking of a tank with soda lye (terrorist attack / strong mechanical stress) – *low probability of occurrence*, *relatively significant impact*;
- The railing's deterioration of leach tanks resulting in accidental fall of a person – *very low probability of production*, *major impact*;
- The damage of treatment plants and water storage facilities (terrorist attack or strong mechanical) – *low probability of occurrence*, *relatively significant impact*;
- Operating errors and/or damage to treatment facilities and industrial water storage – *medium probability*, *low impact*;
- The damage of a tank of ferrous sulphate (terrorist attack or strong mechanical) – *very low probability of producing*, *minor impact*;
- The accidents reagent storage areas – producing *very low probability*, *low impact*;

- The damaging of a lime storage silo (terrorist attack or strong mechanical stress) – *low probability of occurrence*, *relatively significant impact*;
- The failure of the air compressor (buffer vessels explosion/vehicular routes in terms of blocking or failure safety valves) – *low probability of occurrence*, *relatively significant impact*;
- The damage of the fuel system and power distribution (short circuit, overheating etc.) – *medium probability*, eventually fire station has a *major impact*, otherwise the effect is *significant*;
- The curtailment of electricity from external reasons – *extremely low probability*, *major impact*;
- The damage and/or burning of fuel oil above ground tank (terrorist attack or operator error) – *minimum probability of production*, *minor impact*;
- The faults at handling systems for hydrochloric acid solution – *average probability of production*, *minor impact*;
- The crashes of vehicular systems of sodium hydroxide solution – *medium probability*, *low impact*;
- Attempted suicide by ingestion of hydrochloric acid solution – *very low probability of production*, *major impact*;
- Various accidents at work (produced during maintenance and repair or intervention) – *average probability of production*, *significant impact*.

Linguistic variables can be modelled in fuzzy, intuitionistic fuzzy or neutrosophic environments, depending on the research hypothesis or objective. Fuzzy models are appropriate in the case of unavailability of information about the complementary situation, or indeterminacy. When only indeterminacy information is not available the researcher can use the Atanassov models [2].

Every linguistic variable (see above examples) can be described by a degree of truth provided by experts, or computed from explanatory crisp variables. For instance, the overheating can be modelled using temperature over time as explanatory variable. In this case, the degree of truth can be described by mathematical piecewise increasing functions (there is no danger when operating above some limit, but the probability of a risky event increases with temperature).

When a large number of experts will analyse a risky event, some of them agree with an interval of values, and other experts will use another value, or interval of values. When all components: membership, indeterminacy, and non-membership are used then only possible approach is that proposed by Smarandache [17].

For the aim of this paper only fuzzy [22] and Atanassov [2] environments will be used to analyse LCP related risky events.

3. **Risk Analysis using Fuzzy Sets**

Computing with words (CWW) has started with Zadeh's work [22]. This section discusses the usage of CWW paradigm for LCP risk analysis. According to [16], CWW is a "methodology for reasoning, computing and decision-making with information described in natural language", and a "system of

Table 1: TFN parameters for Probability of the risk

No.	Linguistic variable	TFN (a, b, c)
1	absolutely low	(0, 0.01, 0.12)
2	very low	(0.1, 0.125, 0.25)
3	low	(0.2, 0.25, 0.3)
4	fairly low	(0.25, 0.3, 0.4)
5	medium	(0.45, 0.5, 0.55)
6	fairly high	(0.55, 0.625, 0.65)
7	high	(0.75, 0.8, 0.92)
8	very high	(0.9, 0.95, 1.00)
9	absolutely high	(0.99, 0.999, 1.00)

Table 2: Risk Impact TFN parameters

No.	Linguistic variable	TFN (a, b, c)
1	absolutely low	$(0, M/8-\epsilon, M/8)$
2	very low	$(M/8-\epsilon, M/8, M/8+\epsilon)$
3	low	$(M/4-\epsilon, M/4, M/4+\epsilon)$
4	fairly low	$(3M/8-\epsilon, 3M/8, 3M/8+\epsilon)$
5	medium	$(M/2-\epsilon, M/2, M/2+\epsilon)$
6	fairly high	$(5M/8-\epsilon, 5M/8, 5M/8+\epsilon)$
7	high	$(3M/4-\epsilon, 3M/4, 3M/4+\epsilon)$
8	very high	$(7M/8-\epsilon, 7M/8, 7M/8+\epsilon)$
9	absolutely high	$(M-\epsilon, M-\epsilon/2, M)$

computation which adds to traditional systems of computation two important capabilities": a) using the meaning of words and propositions in natural language; b) the reason and compute with words and propositions.

This section describes the Pei & Shi's approach [16] to be used in LCP's risk management. In this view "risk analysis based on CWW belongs to qualitative methods". For computational reason every linguistic variable is described by a fuzzy number. According to [19], a fuzzy set S of a universe X is given by a normalized membership function ϕ_S having bounded support with closed intervals for α-cuts. S is a fuzzy number if X is the set of real numbers, the membership function is convex, and $\phi_S(x) = 1$ for only one element $x \in X$. Computing with fuzzy numbers is based on the representation theorem and the extension principle. If A and B are fuzzy numbers defined by the membership functions ϕ_A, and ϕ_B, and "*" is an operator (addition, subtraction, division, and multiplication), then

$$\phi_{A*B}(z) = \sup_{z=x*y} min\{\phi_A(x), \phi_B(y)\}.$$

In the following the case of triangular fuzzy numbers (TFNs) is considered. The described methodology is valid also for other models as reader easily can discover.

Let A be a TFN given by three numbers $a < b < c$ and the following rules for membership function: if $x \leq a$ then $\phi_A(x) = 0$, if $a < x \leq b$ then $\phi_A(x) = (x-a)/(b-a)$, if $b < x \leq c$ then $\phi_A(x) = (c-x)/(c-b)$, and if $x > b$ then $\phi_A(x) = 0$. It is known that by addition, respective subtraction of two TFNs the obtained result is also a TFN. However, this is not the case for multiplication and division, and max and min operations. If A_i is given by $a_i < b_i < c_i$ for $i \in \{1, 2\}$, then $A_1 + A_2$ is defined by $(a_1 + a_2, b_1 + b_2, c_1 + c_2)$, $A_1 - A_2$ is defined by $(a_1 - c_2, b_1 - b_2, c_1 - a_2)$, if A is given by (a, b, c) then $-A$ is defined as $(-c, -b, -a)$. The multiplication of A_1 and A_2 is obtained by the following rules [13]:

1. If $a_1a_2 \leq z \leq b_1b_2$ then $\phi_{A_1A_2}(z) =$
$$\frac{-(a_1b_2 + a_2b_1 - 2a_1a_2) + \sqrt{(a_1b_2 - a_2b_1)^2 + 4(b_1 - c_1)(b_2 - a_2)z}}{2(b_1 - a_1)(b_2 - a_2)}.$$

2. If $b_1b_2 \leq z \leq c_1c_2$ then $\phi_{A_1A_2}(z) =$
$$\frac{-(c_1b_2 + c_2b_1 - 2c_1c_2) - \sqrt{(c_1b_2 - c_2b_1)^2 + 4(b_1 - c_1)(b_2 - c_2)z}}{2(b_1 - a_1)(b_2 - c_2)}.$$

3. $\phi_{A_1A_2}(z) = 0$, otherwise.

In Table 1, there are described the proposed TFN parameters to associated linguistic variable when the probability of a hazard during LCP risk management is considered: absolutely low, very low, low, fairly low, medium, fairly high, high, very high, and absolutely high.

Modelling the linguistic variables associated to the impact generated by failures depends on the financial range considered by the LCP organisation. A general framework is proposed when M is the largest impact declared by an LCP organization. This value depends on every organisation. The interval $[0, M]$ is decomposed in subintervals to be used for membership TFN modelling: $[0, M/8, M/4, 3M/8, M/2, 5M/8, 3M/4, 7M/8, M]$. Let ϵ a parameter useful to describe the TFN membership functions. Its usefulness becomes clear when the Table 2 is considered. However, for normalisation reason the researcher can use $M = 1$. Similar aspects can be applied during intuitionistic fuzzy modelling as described in the next section.

4. **Risk analysis using Atanassov's intuitionistic fuzzy numbers**

This section describes the usage of intuitionistic fuzzy numbers for risk analysis. For the universe of discourse denoted by X, an intuitionistic fuzzy set (IFS) $A \in X$ is characterized by a membership function $\mu_A(.)$ and a non-membership function $\nu_A(.)$, where $\mu_A : X \to [0, 1]$, and $\nu_A : X \to [0, 1]$, as [2, 14] have described. For each point $x \in X$, $\mu_A(x)$ (resp. $\nu_A(x)$) is the degree of membership (resp. non-membership) of $x \in A$, with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. An intuitionistic fuzzy set becomes a fuzzy set if $\nu_A(x) = 0$ for all $x \in A$.

In general, the results obtained using IFS are better when comparing against the pure fuzzy approach. In the following an extension of the methodology given in [7] based on IFS are used for CWW applied to LCP management. A particular model of IFS is used, namely the triangular intuitionistic fuzzy numbers (TIFNs). However, general intuitionistic fuzzy numbers can be used with an increased computational effort. Let be $a' < a_1 < a_2 < a_3 < a''$ the parameters of a given TIFN having the membership (resp. non-membership) function given by the following rules:

1. If $a_1 \leq x \leq a_2$ then $\mu_A(x) = (x - a_1)/(a_2 - a_1)$;
2. If $a_2 \leq x \leq a_3$ then $\mu_A(x) = (a_3 - x)/(a_3 - a_2)$;
3. If $a' \leq x \leq a_2$ then $\nu_A(x) = (a' - x)/(a_2 - a')$;
4. If $a_2 \leq x \leq a''$ then $\nu_A(x) = (x - a'')/(a'' - a_2)$;
5. If $x < a_1$ or $x > a_3$ then $\mu_A(x) = 0$;
6. If $x < a'$ or $x > a''$ then $\nu_A(x) = 1$.

The following arithmetic operations on Triangular Intuitionistic Fuzzy Numbers are necessary to compute the risk when linguistic variables are used (see [15, 14] for theoretical aspects and application to reliability engineering):

1. If $A = (a_1, a_2, a_3; a', a'')$ and $B = (b_1, b_2, b_3; b', b'')$ are TIFNs, then the sequence defined by $(a_1 + b_1, a_2 + b_2, a_3 + b_3; a' + b', a'' + b'')$ describes the TIFN $A \oplus B$;
2. If $A = (a_1, a_2, a_3; a', a'')$ and $B = (b_1, b_2, b_3; b', b'')$ are TIFNs, then the TIFN $A \otimes B$ is described by $(a_1 b_1, a_2 b_2, a_3 b_3; a' b', a'' b'')$.

Table 3 shows the TIFN parameters of "Probability of Failure" to be considered for LCP component risk assessment.

Assume that there is a component C consisting of n subcomponents C_1, C_2, ..., C_n, and each subcomponent is evaluated by two evaluating items "probability of failure", P_i, and "severity of loss – the impact", denoted by S_i. Both P_i and S_i are linguistic variables as shown in previous section, but modelled by Triangular Intuitionistic Fuzzy Numbers: $\tilde{P}_i$, $\tilde{S}_i$, $i = 1, 2, ..., n$. The TIFN corresponding to the total risk of the component C is given by:

$$\tilde{R} = \sum_{i=1}^{n} \tilde{P}_i \otimes \tilde{S}_i.$$

Finally, a defuzzification technique can be used to obtain a crisp estimation. The methodology was applied during LCP risk analysis in the case of one CET Thermoelectric plant and the proposed framework was appreciated by experts.

Table 3: TIFN parameters for Probability of the risk

No.	Linguistic variable	TIFN $(a_1, a_2, a_3; a', a'')$
1	absolutely low	(0, 0.01, 0.12; 0, 0.12)
2	very low	(0.1, 0.125, 0.25; 0.05, 0.3)
3	low	(0.2, 0.25, 0.3; 0.15, 0.35)
4	fairly low	(0.25, 0.3, 0.4; 0.20, 0.45)
5	medium	(0.45, 0.5, 0.55; 0.40, 0.60)
6	fairly high	(0.55, 0.625, 0.65; 0.50, 0.70)
7	high	(0.75, 0.8, 0.92; 0.70, 0.92)
8	very high	(0.9, 0.95, 1.00; 0.90, 1.00)
9	absolutely high	(0.99, 0.999, 1.00; 0.95, 1.00)

5. **Conclusions**

The paper has presented a theoretical approach for subjective risk assessment. Both fuzzy (Zadeh's model) and intuitionistic fuzzy (Atanassov's model) were investigated for adequacy to risk assessment/analysis of Large Combustion Plants. The case study of CET Thermoelectric plant risk analysis showed that the usage of triangular intuitionistic fuzzy numbers has a better relevance when comparing against the crisp model.

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