Confluence Between Higher Secondary and Universitary Educational System. Approach Based on Projects in Virtual Space

Luminiţa Dominica Moise
Şcoala Superioară Comercială *Nicolae Kretzulescu*, Bucureşti
Ruxandra Cristea
Şcoala Superioară Comercială *Nicolae Kretzulescu*, Bucureşti

Abstract
To motivate and persuade our students to love Mathematics we have devised extracurricular activities in which we bring to their attention the history of mathematics, the outstanding results in various areas of life and of human knowledge that change human perception of reality or reveal the implications of mathematics in the structure, consistency and rigor of other disciplines and how it traces new directions of research in science. This paper is a dissemination of these activities through an approach based on projects in virtual space.
Keywords: *fractals, LabVIEW; swarm intelligence; mathematical statistics; Riemann function; etwinning.*
ACM/AMS Classification: k.3.1

1. **Foreword**

Education has the task to ensure the unhindered development of the personality, to help students learn to live in a dynamic, continuously transforming world. To motivate and persuade our pupils to love Mathematics we have devised extracurricular activities through which we bring to their attention the history of mathematics, outstanding results in various areas of life and of human knowledge that change human perception of reality and reveal the implications of Mathematics in the structure, consistency and rigor of other disciplines, and how it traces new directions of research in science. The boundaries between various fields of knowledge are diminishing or are close to extinction today, there are unexpected, sometimes spectacular connections, so the mathematical stories can be just as captivating as any story and have the same effect - a love of the characters and themes. To create a natural passage towards the next cycle, the university, we guide those interested towards the study of up-to-date mathematics or science in general such as topology and number theory, fractal geometry and chaos theory or mathematical statistics; hyper polyhedra or hyperdeterminants appear as a natural extension of the notion of secondary curriculum.

Because success is the privilege of those who know how to communicate, to think and to reason effectively, we facilitate teamwork. Since we are moving towards a society of knowledge in which "learning" is becoming a constant throughout life, we offer students lists of themes and a webography, we constitute working groups which develop their teamwork skills and create a strong motivation for what they do. Cyberspace is an environment which supports collaboration, so during the projects undertaken with colleagues from other European countries the distance did not matter.

We illustrate the above using examples from current mathematical topics that were the subject of works done together with students:

- Fractals and Chaos Theory;
- Biomimetics and swarm intelligence;
- Mathematical statistics. Applications in sociology, economics and quantum physics;
- Conjecture in number theory. Riemann's zeta function;
- Mathematics of Planet Earth.

2. **Fractals and Chaos Theory**

Everything that is correct thinking is either mathematics
or susceptible to mathematization
Grigore Moisil

For a rigorous foundation of the theory of fractals we have covered a few more chapters of mathematical analysis.

Let (X, d) a complete metric space.

Definition: *The set $H(X)$ consists of nonempty compact subsets of X.*

Definition: *Let $x \in X$ and $B \in H(X)$.*
$d(x, B) = min\{d(x, y)|y \in B\}$ *is called the distance from x to B.*

Definition: *Let A and B of $H(X)$.*
$d(A, B) = max\{d(x, B)|x \in A\}$ *called distance from A to B.*
$h(A, B) = max\{d(A, B), d(B, A)\}$.

It can be shown that h defines a metric on $H(X)$, called the ***Hausdorff-Pompeiu metric***.

Theorem: *$(H(X), h)$ is a complete metric space.*

Metric space $H(X)$ is the space in which they are fractals.

Definition: *Let (X, d) a metric space. $F : X \to X$ is called contraction if there are $k \in [0, 1)$ such that $d(f(x), f(y)) \leq kd(x, y)$ for all $x, y \in X$.*

Hutchinson Operator

Theorem: *Let (X, d) a complete metric space and $f : X \to X$ a k contraction function. Then:*

1. *f has a unique fixed point u and*
2. *for any $x_0 \in X$, the sequence $f^{(n)}(x_0)$ converges to u.*

Contraction principle (Banach)

We consider $\mathbb{R}^2$ (the Euclidean plane) as a complete metric space with the usual distance (Euclidean). Let n be a fixed natural number (not zero) and let for any $j \in \{1, 2, \ldots n\}$, a contraction $W_j : \mathbb{R}^2 \to \mathbb{R}^2$ having the contraction factor k_j. If A is a subset of $\mathbb{R}^2$, we note with $W_j(A)$ the image of the set A through the function W_j.

Definition: *We define the application (Hutchinson operator):*

$$H : H(\mathbb{R}^2) \to H(\mathbb{R}^2),$$

$$H(A) = W_1(A) \cup W_2(A) \cup \ldots \cup W_n(A).$$

We will note: $H = (W_1, W_2, \ldots, W_n)$.

$(\mathbb{R}^2, W_1, W_2, \ldots, W_n)$ is called iterative function system (IFS).

Theorem: *Hutchinson's operator is a contraction on the complete metric space of the compact plan parts $H(\mathbb{R}^2)$ with the Hausdorff distance. In addition, the contraction factor k is the largest element of the set $\{k_1, k_2, \ldots, k_n\}$.*

Definition: *The fixed point $F \in H(\mathbb{R}^2)$ of the Hutchinson's operator (it exists and it is a unique according to the contraction mapping principle) is called the attractor of the iterative function system (deterministic fractal) and it is the limit of the string $H_n(A)$, for all of $A \in H(\mathbb{R}^2)$.*

These notions, which may seem too technical for students in secondary education can be understood because they are richly illustrated with programs made in the LabVIEW environment, a software package that we have called **Fractall**.

For example, the four geometric transformations recursively applied to a square leading to a fern are:

$$f(x,y) = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.16 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f(x,y) = \begin{bmatrix} 0.85 & 0.04 \\ -0.04 & 0.85 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 1.60 \end{bmatrix}$$

$$f(x,y) = \begin{bmatrix} 0.20 & -0.26 \\ 0.23 & 0.22 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 1.60 \end{bmatrix}$$

$$f(x,y) = \begin{bmatrix} -0.15 & 0.28 \\ 0.26 & 0.24 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.44 \end{bmatrix}$$

In their work, the students presented recent applications of fractal geometry in biology and medicine, for example using fractal methods and techniques for analyzing the morphological and anatomical taxa of some plant species in order to clarify the family systemic review, the fractal dimension being used as an indicator of morpho-anatomical classification.

The initiation in the Chaos Theory is made through a logistic function as the discovery of chaos is related to something as simple as a parabola: $f : [0, 1] \to [0, 1]$, $f(x) = cx(1 - x)$, $c \in (0, 4)$.

Concepts such as string orbit, fixed or periodic points, attracting and repelling fixed points or basin of attraction of an attracting fixed point are illustrated with Fractall package programs. It is also an example for the string′s bifurcations and the understanding of the Feigenbaum′s constants.

Should r_n be that critical value for which the logistic map bifurcate into a 2^n period orbit. Mitchell Feigenbaum noticed that the sequence of successive differences appear to converge geometrically.

$$\delta = lim_{n\to\infty}\frac{r_n - r_{n-1}}{r_{n+1} - r_n} = 4.669\ldots$$

3. Biomimetics and collective intelligence

Biomimetics - a science created not long ago - allows, thanks to XXI century technology, a new relationship between man and nature. As the role of mathematics is to provide tools for modelling some natural phenomena, Mathematics may comprise in a few formulas several geometrical shapes, dynamic structures and even collective intelligence. Intelligent behaviors seen in colonies of ants or termites, swarms of bees, flocks of birds, herds of animals, schools of fish etc. are called Swarm Intelligence[1], name taken by artificial intelligence as well to describe a similar collective behavior of self-organized distributed systems. In their work, students noticed the mathematical formulas of the model, as well as its surprising applications in architecture.

The ***flocking model*** algorithm simulates the movement of a set of objects like a flock of birds. In this model, each entity has its own movement decisions, reacting to decisions made by neighbours according to a few simple rules:

- Separation: to avoid collision with other objects in the vicinity;

$$d(P_x, P_b) \leq d_2 \Rightarrow \bar{v}_{sr} = \sum_{x}^{n} \frac{\overline{\bar{v}_x + \bar{v}_b}}{d(P_x, P_b)}$$

[1]Swarm intelligence is an emerging field of biologically-inspired artificial intelligence based on the behavioral models of social insects such as ants, bees, wasps and termites (Bonabeau, 1999).

- Alignment: the moving direction is according to that of its neighbours

$$d(P_x, P_b) \leq d_1 \cap (P_x, P_b) \geq d_2 \Rightarrow \bar{v}_{ar} = \frac{1}{n}\sum_{x}^{n} \bar{v}_x$$

- Cohesion: alignment to the average position of its neighbours.

$$d(P_x, P_b) \leq d_1 \cap (P_x, P_b) \geq d_2 \Rightarrow \bar{v}_{cr} = \sum_{x}^{n} \overline{(P_x - P_b)}$$

Different versions of the ***flocking model*** algorithm were used in parametric architecture to interactively generate emerging spatial patterns. The reason for selecting herds as a study area is the fascinating mismatch between the simplicity of rules and complexity of results.

4. Mathematical Statistics. Applications in sociology, economics and quantum physics

From the desire to understand the subatomic world, students have gone deeply in the study of mathematical statistics. Along with applications in sociology or economics, they discovered probabilities and mathematical statistics in quantum physics.

Physics has always been the basis for understanding the structure of matter. The core of quantum mechanics is the mathematical formalism, which describes physical phenomena, the quantum model of the atom representing a fundamental graphic change in the appearance of the atomic model, as compared to Bohr′s atom.

A first characteristic of the quantum model of the atom is given by the following statement: the state of the atom is described by mathematical functions, quantum mechanics being the theory that describes the subatomic world based on probabilities. Electrons are distributed in areas around the nucleus called orbitals, which are like clouds: the higher density the cloud has, the greater is the probability of detecting an electron. Max Born showed that waves are in fact probabilities, the probability of discovering an electron in a certain place.

5. Conjecture in number theory. Riemann′s zeta function

Prime numbers have revealed many secrets over time, some submitted in students′ work, but their distribution is a problem still unsolved. Riemann′s hypothesis proposes a solution to this problem by analyzing the zeros of the zeta function $\zeta(s)$, but his claim remains unproven. A thorough understanding of the concepts of Riemann′s statement implies a more extensive knowledge on the analysis of the set of complex numbers, but from what students are taught by secondary curriculum we can only get a first impression of Riemann′s statement that concerns the world of mathematicians and not only, because solving this problem would have consequences in other areas as well.

Starting with a generalized harmonic series, we built a zeta function $\zeta(s)$ for s real number and then, by analogy, we introduce zeta function $\zeta(s)$ on the set of complex numbers. The first connection with prime numbers was given by Euler′s formula and with it you can understand why the study of Riemann zeta function will enlighten the problem of distribution of prime numbers and give a better understanding of the fundamental concepts on which mathematics is built.

$$\zeta(s) = \sum_{n-1}^{\infty} \frac{1}{n^s}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1-p^{-s}}$$

6. **Mathematics of Planet Earth**

All other sciences are often based on mathematics to articulate their findings and assumptions. Today, anyone can be surprised by the announcement of the potential evidence of the existence of Higgs boson at CERN, but this boson could not have been predicted without mathematics. Mathematics is the language of science

Marcus du Sautoy

Man has always wanted to understand the world in which he lives so he sought representations and models for phenomena and processes he observed, out of his desire to know or the need to predict their future evolution. Over time, mathematics has provided better tools designed to enable a better understanding of the world around and to make accurate predictions of its evolution, mathematical modelling being a complex activity through which a certain part of the universe is represented in mathematical symbols.

Mathematical models are presented in a large variety of forms: linear and nonlinear models, deterministic or stochastic, static or dynamic, discrete or continuous.

Although we can give many examples of this sort, mathematical modelling is a difficult concept to define. At a first insight, it is about applied mathematics in physics, chemistry, biology, economics. As AC Fowler stated in his book ***Mathematical Models in the Applied Sciences***, Cambridge University Press in 1998: *There are no rules and no clear understanding of a correct path to be followed in mathematical modelling. Therefore there are* *only a few texts that address this issue in a serious way. Mathematical modelling is learned through practice, through exercises on different examples.* The complexity of a model always involves a balance between the simplicity and the accuracy of its representation.

The complex problems of the contemporary world related to food, housing, water and energy, the evolution of dynamical systems - the atmosphere, the Earth tectonics, the water circuit which often put human lives at risk, all require quick fixes.

Let′s just take a simple example, the symmetry of nature. If crystals can be described by a group of symmetries, quasi-crystals violate the rules of symmetry seen in traditional crystalline structures. They can shape living structures, such as viruses which are often symmetrical in shape and this is the secret why they are so virulent and powerful. A group of scientists have studied the protein of the influenza virus that is involved in the mechanism used by the virus to take control of the processes in the human cells it infects and they found a quasi crystal pattern in these structures. Their mathematical support is the pavement theory, which means covering a plan with a subset, without overlapping, each of its pieces being topologically equivalent to a disc.

Seeking information about mathematical modelling, we found under the name of ***Mathematics of Planet Earth 2013*** large events under the auspices of UNESCO, carried out by large organizations that aim to provide information, from simple to complex, of the way in which mathematics is involved in solving the problems of the contemporary world.

7. Conclusion

Through extra-curricular activities, we wanted that the students understand how mathematics is written today, to live the history of mathematics at the present time.

Finally, because we started this paper with fractal geometry, a reference field in mathematics of the end of the twentieth century, we would like to bring as argument for our efforts some thoughts expressed in the granting the title of Doctor Honoris Causa of the University of St Andrews to Benoit Mandelbrot - the father of fractal geometry:

. . . at the end of a century (the twentieth century) when the concept of intellectual, political and moral human progress is regarded as ambiguous and equivocal at best case, there is at least one area of human activity where the idea of real progress and its fulfilment are not ambiguous but crystal clear. This field is mathematics.

References

1. Gica A., Panaitopol L., *Probleme celebre de teoria numerelor*, Editura Universităţii Bucureşti, 1998
2. W.G.Chinn, N. E. Steenrod, *Introducere în topologie*, Editura Tehnică, Bucureşti, 1981

3. Michael F.Barnsley, *Fractals everywhere*, Second Edition, Academic Press Professional, 1993.
4. Heinz Otto Peitgen, Harmut Jurgens, Dietmar Saupe, *Chaos and New frontiers of science*, Springer Verlag 1992
5. Tom Savu, Neacşu Ion, Grigorescu Ştefan, Garabet Elena Mihaela, *Bazele instrumentaţiei virtuale LabView*, Editura Atelier didactic, Bucureşti, 2006
6. Robert L. Devamy, *Chaos, Fractals and Dynamics*, Wesley Publishing Company, 1990.
7. Nicolescu Anamaria Carmen, Andrei Marin, *Fractal Analysis Of Endemic Plants From Romanian Carpathians*, FARMACIA, 2011, Vol. 59, 4.
8. www.mpe2013.org